Advection-based Function Matching on Surfaces

Abstract:
A tangent vector field on a surface is the generator of a smooth family of maps from the surface to itself, known as the flow. Given a scalar function on the surface, it can be transported, or advected, by composing it with a vector field's flow. Such transport is exhibited by many physical phenomena, e.g., in fluid dynamics. In this paper, we are interested in the inverse problem: given source and target functions, compute a vector field whose flow advects the source to the target. We propose a method for addressing this problem, by minimizing an energy given by the advection constraint together with a regularizing term for the vector field. Our approach is inspired by a similar method in computational anatomy, known as LDDMM, yet leverages the recent framework of functional vector fields for discretizing the advection and the flow as operators on scalar functions. The latter allows us to efficiently generalize LDDMM to curved surfaces, without explicitly computing the flow lines of the vector field we are optimizing for. We show two approaches for the solution: using linear advection with multiple vector fields, and using non-linear advection with a single vector field. We additionally derive an approximated gradient of the corresponding energy, which is based on a novel vector field transport operator. Finally, we demonstrate applications of our machinery to intrinsic symmetry analysis, function interpolation and map improvement.