Affine Springer fibers and depth zero $L$-packets.

Abstract:

Let $G$ be a connected reductive group over $F=F_q((t))$, splitting over a maximal unramified extension. To every tamely ramified Langlands parameter $\lambda$ in general position gives rise to a finite set $\pi_{\lambda}$ of irreducible admissible representations of $G(F)$, called the $L$-packet.

The goal of this talk is to provide a geometric description of characters $\chi_{\pi}$ of all $\pi \in \pi_{\lambda}$ in terms of homology of affine Springer fibers. As an application, we give a geometric proof of the stability of sum $\chi_{\lambda}^{st}:=\sum_{\pi \in \pi_{\lambda}} \chi_{\pi}$. Furthermore, we show that the $\chi_{\lambda}^{st}$'s are compatible with inner twistings.

This is a joint work with Roman Bezrukavnikov (https://arxiv.org/abs/2104.13123), and is a first step in a joint outgoing project of the two of us with David Kazhdan, whose goal is to obtain similar results for general depth zero representations.