Around the Thom-Sebastiani theorem

Abstract:

For germs of holomorphic functions \( f : \mathbf{C}^{m+1} \to \mathbf{C} \), \( g : \mathbf{C}^{n+1} \to \mathbf{C} \) having an isolated critical point at 0 with value 0, the classical Thom-Sebastiani theorem describes the vanishing cycles group \( \Phi^{m+n+1}(f \oplus g) \) (and its monodromy) as a tensor product \( \Phi^m(f) \otimes \Phi^n(g) \), where \((f \oplus g)(x,y) = f(x) + g(y), x = (x_0,\ldots,x_m), y = (y_0,\ldots,y_n)\). I will discuss algebraic variants and generalizations of this result over fields of any characteristic, where the tensor product is replaced by a certain local convolution product, as suggested by Deligne. The main theorem is a Künneth formula for \( R\Psi \) in the framework of Deligne's theory of nearby cycles over general bases.