Automorphy in string scattering and small representations

I will review how automorphic representations arise in the calculation of string theory scattering amplitudes. As follows from the work of Green, Miller, Vanhove, Pioline and others, the automorphic representations are associated with split real groups of a certain exceptional family. In the cases that are well understood, these representations have very small Gelfand-Kirillov dimension. Their Fourier expansion can be calculated using different methods and confirms physical expectation on the wavefront set. In work with Gourevitch, Gustafsson, Persson and Sahi, the method of Whittaker pairs was employed to systematize this analysis. I will also comment on the cases that are less well understood in physics and that appear to go beyond the standard notion of automorphic representations since the usual $\mathbb{Z}$-finiteness condition is violated.