Balancing vectors in any norm

Abstract:

In the vector balancing problem, we are given \( N \) vectors \( v_1, \ldots, v_N \) in an \( n \)-dimensional normed space, and our goal is to assign signs to them, so that the norm of their signed sum is as small as possible. The balancing constant of the vectors is the smallest number \( \beta \), such that any subset of the vectors can be balanced so that their signed sum has norm at most \( \beta \).

The vector balancing constant generalizes combinatorial discrepancy, and is related to rounding problems in combinatorial optimization, and to the approximate Caratheodory theorem. We study the question of efficiently approximating the vector balancing constant of any set of vectors, with respect to an arbitrary norm. We show that the vector balancing constant can be approximated in polynomial time to within factors logarithmic in the dimension, and is characterized by (an appropriately optimized version of) a known volumetric lower bound. Our techniques draw on results from geometric functional analysis and the theory of Gaussian processes. Our results also imply an improved approximation algorithm for hereditary discrepancy.

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