Catalan Numbers and Labelled Graphs

Abstract:

The Catalan numbers form a sequence of integers $C_t$. A collection of sets $H_t$ with $|H_t| = C_t$ for all $t$ is called a Catalan set. Many examples of Catalan sets are known; the triangulations of the $(t+2)$-gon, the Dyck paths from $(0,0)$ to $(0, 2t)$ and the nilpotent ideals in the Borel subalgebra of $\mathfrak{sl}_t$ to name but a few. In my talk I will present a new example of a Catalan set, which has a remarkable property: for all $t$, $H_t$ decomposes into a (non-disjoint) union of $C_{t-1}$ distinct subsets each of cardinality $2^{t-1}$. Moreover, one may define certain interesting labelled graphs for $H_t$ and obtain the above decomposition in a natural way. The subgraphs corresponding to the aforementioned subsets are labelled hypercubes with some edges missing. The motivation of this work was the study of the additive structure of the Kashiwara crystal $\mathcal{B}(\infty)$. 