Computational anatomy and splines on manifolds

Abstract:

Computational anatomy considers spaces of shapes (e.g. medical images) endowed with a Riemannian metric (Sobolev type). The area blends techniques from differential geometry (geometric mechanics), analysis and statistics. The EPDiff equation, which is basically an extension of Euler's equation, without the incompressibility assumption, is often used to match shapes. Time-varying images (4DCA), one of the current research themes in the area. In longitudinal studies (say for Alzheimer's disease) snapshots at given times are interpolated/regressed. The problem arises of comparing two such sequences for classification purposes. For some background, see the recent workshop http://www.mat.univie.ac.at/~shape2015/schedule.html.

In this talk we discuss finite dimensional examples using landmarks. A short process is interpreted as a tangent vector in the space of images. This leads to control problems whose state space is a tangent bundle. Usually, cubic Riemannian splines are taken, ie., minimizing the $L^2$ norm of the acceleration vector, for paths connecting two tangent vectors under a fixed time. We propose as an alternative to cubic splines the time minimal problem under bounded acceleration (morally the $L^\infty$ norm). We suggest that both splines problems on $T^*\left(T^*S^2\right)$ are completely integrable in the Arnold-Liouville sense. Along the way, we present general technical results about the underlying symplectic structures of control problems whose state space has a bundle structure.

This is joint ongoing work with Paula Balseiro, Alejandro Cabrera, and Teresa Stuchi.