The theory of locally compact quantum groups grew out of the need to extend Pontryagin’s duality for locally compact abelian groups to a wider class of objects, as well as from a modern “quantum” point of view suggesting the replacement of some algebras of functions on a group by non-commutative objects, namely operator algebras. In this talk, which will be split into two parts, we will show how several fundamental notions from probability and geometric group theory fit in this framework.

The first part will be an introduction to locally compact quantum groups. We will present the rationale and the definitions, give examples, and explain how the theory is related to other branches of math. If time permits, we will also touch upon more specific notions related to the second part.

In the second part we will discuss convolution semigroups of states, as well as generating functionals, on locally compact quantum groups. One type of examples comes from probability: the family of distributions of a Lévy process form a convolution semigroup, which in turn admits a natural generating functional. Another type of examples comes from (locally compact) group theory, involving semigroups of positive-definite functions and conditionally negative-definite functions, which provide important information about the group’s geometry. We will explain how these notions are related and how all this extends to the quantum world; derive geometric characterizations of two approximation properties of locally compact quantum groups; see how generating functionals may be (re)constructed and study their domains; and indicate how our results can be used to study cocycles.

Based on joint work with Adam Skalski.

No background in operator algebras will be assumed.