Counting solutions of differential equations

Abstract:

We consider the following problem: given a set of algebraic conditions on an \( n \)-tuple of functions and their first \( l \) derivatives, admitting finitely many solutions (in a differentiably closed field), can one give an upper bound for the number of solutions?

I will present estimates in terms of the degrees of the algebraic conditions, or more generally the volumes of their Newton polytopes (analogous to the Bezout and BKK theorems). The estimates are singly-exponential with respect to \( n,l \) and have the natural asymptotic with respect to the degrees or Newton polytopes, sharpening previous doubly-exponential estimates due to Hrushovski and Pillay. I will also discuss some diophantine applications to counting transcendental lattice points on algebraic varieties.