The Danzer problem and a solution to a problem of Gowers

Abstract:

Is there a point set \( Y \) in \( \mathbb{R}^d \), and \( C>0 \), such that every convex set of volume 1 contains at least one point of \( Y \) and at most \( C \)? This discrete geometry problem was posed by Gowers in 2000, and it is a special case of an open problem posed by Danzer in 1965. I will present two proofs that answers Gowers' question with a NO. The first approach is dynamical; we introduce a dynamical system and classify its minimal subsystems. This classification in particular yields the negative answer to Gowers' question. The second proof is direct and it has nice applications in combinatorics. The talk will be accessible to a general audience. [This is a joint work with Omri Solan and Barak Weiss].