A description of two-generated subalgebras of a polynomial ring in one variable and a new proof of the AMS theorem

Abstract:

The famous AMS (Abhyankar-Moh-Suzuki) theorem states that if two polynomials \( f \) and \( g \) in one variable with coefficients in a field \( F \) generate all algebra of polynomials, i.e. any polynomial \( h \) in one variable can be expressed as \( h = H(f, g) \) where \( H \) is a polynomial in two variables, then either the degree of \( f \) divides the degree of \( g \), or the degree of \( g \) divides the degree of \( f \), or the degree of \( f \) and the degree of \( g \) are divisible by the characteristic of the field \( F \). There were several wrong published proofs of this theorem and there are many correct published proofs of this theorem but all of them either long or not self-contained. Recently I found a (relatively) short and self-contained proof which is not published yet and which I can explain in one-two hours.