Discontinuity of the phase transition for the planar random-cluster and Potts models with $q > 4$

Abstract:

The random-cluster model is a dependent percolation model where the weight of a configuration is proportional to $q$ to the power of the number of connected components. It is highly related to the ferromagnetic $q$-Potts model, where every vertex is assigned one of $q$ colors, and monochromatic neighbors are encouraged. Through non-rigorous means, Baxter showed that the phase transition is first-order whenever $q > 4$ - i.e. there are multiple Gibbs measures at criticality. We provide a rigorous proof of this claim. Like Baxter, our proof uses the correspondence between the above models and the six-vertex model, which we analyze using the Bethe ansatz and transfer matrix techniques. We also prove Baxter’s formula for the correlation length of the models at criticality.

This is joint work with Hugo Duminil-Copin, Maxime Gangebin, Ioan Manolescu, and Vincent Tassion.