The evolution, through spatially periodic linear dispersion, of rough initial data leads to surprising quantized structures at rational times, and fractal, non-differentiable profiles at irrational times. The Talbot effect, named after an optical experiment by one of the founders of photography, was first observed in optics and quantum mechanics, and leads to intriguing connections with exponential sums arising in number theory. Ramifications of these phenomena and recent progress on the analysis, numerics, and extensions to nonlinear wave models will be discussed.