David Jerison (MIT) and Ron Rosenthal (Technion).

DOUBLE TALK

Abstract:

David Jerison: Localization of eigenfunctions via an effective potential
We discuss joint work with Douglas Arnold, Guy David, Marcel Filoche and Svitlana Mayboroda. Consider the Neumann boundary value problem for the operator $\mathcal{L} u = \nabla \cdot (A \nabla u) + Vu$ on a Lipschitz domain $\Omega$ and, more generally, on a manifold with or without boundary. The eigenfunctions of $\mathcal{L}$ are often localized, as a result of disorder of the potential $V$, the matrix of coefficients $A$, irregularities of the boundary, or all of the above. In earlier work, Filoche and Mayboroda introduced the function $u$ solving $\mathcal{L} u = 1$, and showed numerically that it strongly reflects this localization. Here, we deepen the connection between the eigenfunctions and this “landscape” function $u$ by proving that its reciprocal $1/u$ acts as an “effective potential”. The effective potential governs the exponential decay of the eigenfunctions of the system and delivers information on the distribution of eigenvalues near the bottom of the spectrum.

Ron Rosenthal: Eigenvector correlation in the complex Ginibre ensemble
The complex Ginibre ensemble is a non-Hermitian random matrix on $\mathbb{C}^N$ with i.i.d. complex Gaussian entries normalized to have mean zero and variance $1/N$. Unlike the Gaussian unitary ensemble, for which the eigenvectors are orthogonal, the geometry of the eigenbases of the Ginibre ensemble are not particularly well understood. We will discuss some results regarding the analytic and algebraic structure of eigenvector correlations in this matrix ensemble. In particular, we uncover an extended algebraic structure which describes the asymptotic behavior (as $N$ goes to infinity) of these correlations. Our work extends previous results of Chalker and Mehlig [CM98], in which the correlation for pairs of eigenvectors was computed. Based on a joint work with Nick Crawford.