Eigenvalue bounds on surfaces: some recent advances

We will give an overview of some recent results on Laplace and Steklov eigenvalue estimates on Riemannian surfaces. In particular, we will present an upper bound on the first Laplace eigenvalue for non-orientable surfaces, extending some classical inequalities due to Yang, Li and Yau. We will also discuss the Steklov eigenvalue problem that has attracted a lot of attention in the past decade. In particular, geometric estimates on Steklov eigenvalues of arbitrary index will be presented.