Eigenvalue confinement and spectral gap for random simplicial complexes

Abstract:

We consider the adjacency operator of the Linial-Meshulam model for random simplicial complexes on \( n \) vertices, where each \( d \)-cell is added independently with probability \( p \) to the complete \( (d-1) \)-skeleton. From the point of view of random matrix theory, the adjacency matrix is a sparse, self-adjoint random matrix with dependent entries. Under the assumption \( np(1-p) >> \log^4 n \), we prove that the spectral gap between the \( \binom{n-1}{d} \) smallest eigenvalues and the remaining \( \binom{n-1}{d-1} \) eigenvalues is \( np - 2\sqrt{dnp(1-p)}(1+o(1)) \) with high probability. This estimate follows from a more general result on eigenvalue confinement. In addition, we prove that the global distribution of the eigenvalues is asymptotically given by the semicircle law. Based on a joint work with Antti Knowles.