Eigenvalue confinement and spectral gap for random simplicial complexes

Abstract:
We consider the adjacency operator of the Linial-Meshulam model for random simplicial complexes on $n$ vertices, where each $d$-cell is added independently with probability $p$ to the complete $(d-1)$-skeleton. From the point of view of random matrix theory, the adjacency matrix is a sparse, self-adjoint random matrix with dependent entries. Under the assumption $np(1-p) \gg \log^4 n$, we prove that the spectral gap between the $\binom{n-1}{d}$ smallest eigenvalues and the remaining $\binom{n-1}{d-1}$ eigenvalues is $np - 2\sqrt{dnp(1-p)}(1+o(1))$ with high probability. This estimate follows from a more general result on eigenvalue confinement. In addition, we prove that the global distribution of the eigenvalues is asymptotically given by the semicircle law. Based on a joint work with Antti Knowles.