Eigenvalue confinement and spectral gap for random simplicial complexes

Abstract:

We consider the adjacency operator of the Linial-Meshulam model for random simplicial complexes on \(n\) vertices, where each \(d\)-cell is added independently with probability \(p\) to the complete \((d-1)\)-skeleton. From the point of view of random matrix theory, the adjacency matrix is a sparse, self adjoint random matrix with dependent entries. Under the assumption \(np(1-p)\gg \log^4 n\), we prove that the spectral gap between the \(\binom{n-1}{d}\) smallest eigenvalues and the remaining \(\binom{n-1}{d-1}\) eigenvalues is \(np-2\sqrt{dnp(1-p)}(1+o(1))\) with high probability. This estimate follows from a more general result on eigenvalue confinement. In addition, we prove that the global distribution of the eigenvalues is asymptotically given by the semicircle law. Based on a joint work with Antti Knowles.