On exceptional symmetric Gelfand pairs

Abstract:

A pair of a reductive linear algebraic group \( G \) and a subgroup \( H \) is said to be a Gelfand pair when, roughly speaking, the representation of \( G \) on \( \mathcal{C}^\infty(G/H) \) is multiplicity free. Symmetric pairs, those where \( H \) is the fixed-point set of an involution on \( G \), give many examples of Gelfand pairs. The Aizenbud-Gourevitch criterion, based on a previous distributional criterion by Gelfand and Kazhdan, was introduced to prove that many classical symmetric pairs are Gelfand pairs. For complex symmetric pairs, it says that the Gelfand property holds if the pair and all its descendants (centralizers of admissible semisimple elements) satisfy a certain regularity condition (expressed in terms of invariant distributions). In this talk we will focus on the twelve exceptional complex symmetric pairs and combine the Aizenbud-Gourevitch criterion with Lie-theoretic techniques. We will first introduce the concept of a pleasant pair, which will allow us to prove regularity for many pairs. We will then show how to compute descendants visually, thanks to the Satake diagram. The combination of these results with the criterion yields that nine out of the twelve pairs are Gelfand, and that the Gelfand property for the remaining three is equivalent to the regularity of one exceptional and two classical symmetric pairs.