Explicit Binary Tree Codes with Polylogarithmic Size Alphabet

Abstract:

This paper makes progress on the problem of explicitly constructing a binary tree code with constant distance and constant alphabet size.

For every constant \( \delta < 1 \) we give an explicit binary tree code with distance \( \delta \) and alphabet size \( \text{poly}(\log n) \), where \( n \) is the depth of the tree. This is the first improvement over a two-decade-old construction that has an exponentially larger alphabet of size \( \text{poly}(n) \).

As part of the analysis, we prove a bound on the number of positive integer roots a real polynomial can have in terms of its sparsity with respect to the Newton basis—a result of independent interest.

Joint work with G. Cohen and B. Haeupler