Extremal individuals in branching systems

Abstract:

Branching processes have been subject of intense and fascinating studies for a long time. In my talk I will present two problems in order to highlight their rich structure and various technical approaches in the intersection of probability and analysis.

Firstly, I will present results concerning a branching random walk with the time-inhomogeneous branching law. We consider a system of particles, which at the end of each time unit produce offspring randomly and independently. The branching law, determining the number and locations of the offspring is the same for all particles in a given generation. Until recently, a common assumption was that the branching law does not change over time. In a pioneering work, Fang and Zeitouni (2010) considered a process with two macroscopic time intervals with different branching laws. In my talk I will present the results when the branching law varies at mesoscopic and microscopic scales. In arguably the most interesting case, when the branching law is sampled randomly for every step, I will present a quenched result with detailed asymptotics of the maximal particle. Interestingly, the disorder has a slowing-down effect manifesting itself on the log level.

Secondly, I will turn to the classical branching Brownian motion. Let us assume that particles move according to a Brownian motion with drift \( \mu \) and split with intensity 1. It is well-know that for \( \mu \geq 2\sqrt{\mu} \) the system escapes to infinity, thus the overall minimum is well-defined. In order to understand it better, we modify the process such that the particles are absorbed at position 0. I will present the results concerning the law of the number of absorbed particles \( N \). In particular I will concentrate on \( P(N=0) \) and the maximal exponential moment of \( N \). This reveals new deep connections with the FKPP equation. Finally, I will also consider \( -2\sqrt{\mu} < \mu < 2\sqrt{\mu} \) and \( N_{xt} \) the number of particles absorbed until the time \( t \) when the system starts from \( x \). In this case I will show the convergence to the traveling wave solution of the FKPP equation for an appropriate choice of \( x, t \rightarrow \infty \).

The results were obtained jointly with B. Mallein and with J. Berestycki, E. Brunet and S. Harris respectively.