



THE WEIZMANN INSTITUTE OF SCIENCE  
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE  
Geometric Functional Analysis and Probability Seminar

Room 155 ,Ziskind Building  
on Thursday, Feb 14, 2019  
at 13:30

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WIS

On the extremal particles of a Coulomb gas and random polynomials

Abstract:

The purpose of this talk is to understand the behavior of the extremal zeros of random polynomials of the form  $P_N(z) = \sum_{k=0}^N a_k R_k(z)$  where the family  $(R_k)_{k \leq N}$  is an orthonormal basis for the scalar product  $\langle P, Q \rangle = \int P(z) \overline{Q(z)} e^{-2N V^{\nu}(z)} d\nu(z)$  with  $\nu$  a radial probability measure on  $\mathbb{C}$  and  $V^{\nu}(z) = \int \log |z-w| d\nu(w)$ .

Although the zeros of these polynomials spread like the measure  $\nu$ , the zeros of maximum modulus lie outside of the support. More precisely, the point process of the roots outside of the support of the equilibrium measure converges towards the Bergman point process of the complement of the support.

We also study similar results on a model of Coulomb gases in dimension 2 where the confining potential is generated by the presence of a fixed background of positive charges. If  $\nu$  is a probability measure, we study the system of particles with joint distribution on  $\mathbb{C}^N$ ,  $\frac{1}{Z_N} \prod_{i \leq j} |x_i - x_j|^2 e^{-2(N+1) \sum_{k=1}^N V^{\nu}(x_k)} d\ell_{\mathbb{C}^N}(x_1, \dots, x_N)$ . This model is closely related to the study of the zeros of random polynomials. We show that the extremal particles of this Coulomb gas present a similar behavior to the random polynomial setting.

All the results mentioned above are done in collaboration with David Garcia-Zelada.