The frog model on trees

Imagine that every vertex of a graph contains a sleeping frog. At time 0, the frog at some designated vertex wakes up and begins a simple random walk. When it lands on a vertex, the sleeping frog there wakes up and begins its own simple random walk, which in turn wakes up any sleeping frogs it lands on, and so on. This process is called the frog model.

I’ll talk about a question posed by Serguei Popov in 2003: On an infinite d-ary tree, is the frog model recurrent or transient? That is, is each vertex visited infinitely or finitely often by frogs? The answer is that it depends on d: there’s a phase transition between recurrence and transience as d grows. Furthermore, if the system starts with Pois(m) sleeping frogs on each vertex independently, there’s a phase transition as m grows. This is joint work with Christopher Hoffman and Matthew Junge.