מכון ויצמן למדע

# THE WEIZMANN INSTITUTE OF SCIENCE

## FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

### Geometric Functional Analysis and Probability Seminar

Room 155 ,Ziskind Building
on Thursday, Jun 14, 2018
at 13:30

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Technion

## The Gaussian Double-Bubble and Multi-Bubble Conjectures

Abstract:

The classical Gaussian isoperimetric inequality, established in the 70's independently by Sudakov-Tsirelson and Borell, states that the optimal way to decompose $\mathbb{R}^n$ into two sets of prescribed Gaussian measure, so that the (Gaussian) area of their interface is minimal, is by using two complementing half-planes. This is the Gaussian analogue of the classical Euclidean isoperimetric inequality, and is therefore referred to as the "single-bubble" case.

A natural generalization is to decompose $\mathbb{R}^n$ into $q \geq 3$ sets of prescribed Gaussian measure. It is conjectured that when $q \leq n+1$, the configuration whose interface has minimal (Gaussian) area is given by the Voronoi cells of $q$ equidistant points. For example, for $q=3$ (the "double-bubble"conjecture) in the plane ($n=2$), the interface is conjectured to be a "tripod" or "Y" - three rays meeting at a single point in 120 degree angles. For $q=4$ (the "triple-bubble" conjecture) in $\mathbb{R}^3$, the interface is conjectured to be a tetrahedral cone.

We confirm the Gaussian double-bubble and, more generally, multi-bubble conjectures for all $3 \leq q \leq n+1$. The double-bubble case $q=3$ is simpler, and we will explain why.

None of the numerous methods discovered over the years for establishing the classical $q=2$ case seem amenable to the $q \geq 3$ cases, and our method consists of establishing a Partial Differential Inequality satisfied by the isoperimetric profile. To treat $q > 3$, we first prove that locally minimimal configurations must have flat interfaces, and thus convex polyhedral cells. Uniqueness of minimizers up to null-sets is also established.

This is joint work with Joe Neeman (UT Austin).

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