Gel'fand-Kirillov Dimension of Algebras: Prime Spectra, Gradations and Radicals

Abstract:

We study properties of affine algebras with small Gel'fand-Kirillov dimension, from the points of view of the prime spectrum, gradations and radical theory.

As an application, we are able to prove that \( \mathbb{Z} \)-graded algebras with quadratic growth, and graded domains with cubic growth have finite (and efficiently bounded) classical Krull dimension; this is motivated by Artin's conjectured geometric classification of non-commutative projective surfaces, and by opposite examples in the non-graded case.

As another application, we prove a graded version of a dichotomy question raised by Braun and Small, between primitive algebras (namely, algebras admitting faithful irreducible representations) and algebras satisfying polynomial identities.

If time permits, we discuss approximations of the well-studied Koethe problem and in particular prove a stability result for certain radicals under suitable growth conditions.

We finally propose further questions and possible directions, which already stimulated new constructions of monomial algebras.

This talk is partially based on a joint work with A. Leroy, A. Smoktunowicz and M. Ziembowski.