Generalized Harish-Chandra functors for general linear groups over finite local rings

Abstract:

Let K be a commutative ring. Consider the groups GLn(K). Bernstein and Zelevinsky have studied the representations of the general linear groups in case the ring K is a finite field. Instead of studying the representations of GLn(K) for each n separately, they have studied all the representations of all the groups GLn(K) simultaneously. They considered on R := nR(GLn(K)) structures called parabolic (or Harish-Chandra) induction and restriction, and showed that they enrich R with a structure of a so-called positive self-adjoint Hopf algebra (or PSH algebra). They use this structure to reduce the study of representations of the groups GLn(K) to the following two tasks:

1. Study a special family of representations of GLn(K), called cuspidal representations. These are representations which do not arise as direct summands of parabolic induction of smaller representations.
2. Study representations of the symmetric groups. These representations also have a nice combinatorial description, using partitions.

In this talk I will discuss the study of representations of GLn(K) where K is a finite quotient of a discrete valuation ring (such as Z/qr or k[[x]]/xr, where k is a finite field). One reason to study such representations is that all continuous complex representations of the groups GLn(Zp) and GLn(k[[x]]) (where Zp denotes the p-adic integers) arise from these finite quotients. I will explain why the natural generalization of the Harish-Chandra functors do not furnish a PSH algebra in this case, and how is this related to the Bruhat decomposition and Gauss elimination. In order to overcome this issue we have constructed a generalization of the Harish-Chandra functors. I will explain this generalization, describe some of the new functors properties, and explain how can they be applied to studying complex representations.

The talk will be based on a joint work with Tyrone Crisp and Uri Onn.