Generalized Harish-Chandra functors for general linear groups over finite local rings

Abstract:

Let \( K \) be a commutative ring. Consider the groups \( \text{GL}_n(K) \). Bernstein and Zelevinsky have studied the representations of the general linear groups in case the ring \( K \) is a finite field. Instead of studying the representations of \( \text{GL}_n(K) \) for each \( n \) separately, they have studied all the representations of all the groups \( \text{GL}_n(K) \) simultaneously. They considered on \( R := n\mathcal{R}(\text{GL}_n(K)) \) structures called parabolic (or Harish-Chandra) induction and restriction, and showed that they enrich \( R \) with a structure of a so-called positive self-adjoint Hopf algebra (or PSH algebra). They use this structure to reduce the study of representations of the groups \( \text{GL}_n(K) \) to the following two tasks:

1. Study a special family of representations of \( \text{GL}_n(K) \), called cuspidal representations. These are representations which do not arise as direct summands of parabolic induction of smaller representations.

2. Study representations of the symmetric groups. These representations also have a nice combinatorial description, using partitions.

In this talk, I will discuss the study of representations of \( \text{GL}_n(K) \) where \( K \) is a finite quotient of a discrete valuation ring (such as \( Z = \mathbb{Z}/p\mathbb{Z} \) or \( k[x] = \mathbb{k}[x] \), where \( k \) is a finite field). One reason to study such representations is that all continuous complex representations of the groups \( \text{GL}_n(\mathbb{Z}_p) \) and \( \text{GL}_n(k[[x]]) \) (where \( \mathbb{Z}_p \) denotes the \( p \)-adic integers) arise from these finite quotients. I will explain why the natural generalization of the Harish-Chandra functors do not furnish a PSH algebra in this case, and how is this related to the Bruhat decomposition and Gauss elimination. In order to overcome this issue, we have constructed a generalization of the Harish-Chandra functors. I will explain this generalization, describe some of the new functors' properties, and explain how they can be applied to studying complex representations.

The talk will be based on a joint work with Tyrone Crisp and Uri Onn.