Generalized Harish-Chandra functors for general linear groups over finite local rings

Abstract:

Let K be a commutative ring. Consider the groups GLn(K). Bernstein and Zelevinsky have studied the representations of the general linear groups in case the ring K is a finite eld. Instead of studying the representations of GLn(K) for each n separately, they have studied all the representations of all the groups GLn(K) simultaneously. They considered on $R := nR(GLn(K))$ structures called parabolic (or Harish-Chandra) induction and restriction, and showed that they enrich R with a structure of a so called positive self adjoint Hopf algebra (or PSH algebra). They use this structure to reduce the study of representations of the groups GLn(K) to the following two tasks:
1. Study a special family of representations of GLn(K), called cuspidal representa- tions. These are representations which do not arise as direct summands of parabolic induction of smaller representations.
2. Study representations of the symmetric groups. These representation also has a nice combinatorial description, using partitions.

In this talk I will discuss the study of representations of GLn(K) where K is a finite quotient of a discrete valuation ring (such as $Z=pr$ or $k[x]=xr$, where k is a finite eld). One reason to study such representation is that all continuous complex representations of the groups GLn(Zp) and GLn(k[[x]]) (where Zp denotes the p-adic integers) arise from these finite quotients. I will explain why the natural generalization of the Harish-Chandra functors do not furnish a PSH algebra in this case, and how is this related to the Bruhat decomposition and Gauss elimination. In order to overcome this issue we have constructed a generalization of the Harish-Chandra functors. I will explain this generalization, describe some of the new functors properties, and explain how can they be applied to studying complex representations.

The talk will be based on a joint work with Tyrone Crisp and Uri Onn.