Generalized Harish-Chandra functors for general linear groups over finite local rings

Abstract:

Let $K$ be a commutative ring. Consider the groups $\text{GL}_n(K)$. Bernstein and Zelevinsky have studied the representations of the general linear groups in case the ring $K$ is a finite field. Instead of studying the representations of $\text{GL}_n(K)$ for each $n$ separately, they have studied all the representations of all the groups $\text{GL}_n(K)$ simultaneously. They considered on $R := nR(\text{GL}_n(K))$ structures called parabolic (or Harish-Chandra) induction and restriction, and showed that they enrich $R$ with a structure of a so called positive self adjoint Hopf algebra (or PSH algebra). They use this structure to reduce the study of representations of the groups $\text{GL}_n(K)$ to the following two tasks:

1. Study a special family of representations of $\text{GL}_n(K)$, called cuspidal representations. These are representations which do not arise as direct summands of parabolic induction of smaller representations.
2. Study representations of the symmetric groups. These representation also has a nice combinatorial description, using partitions.

In this talk I will discuss the study of representations of $\text{GL}_n(K)$ where $K$ is a finite quotient of a discrete valuation ring (such as $\mathbb{Z} = \mathbb{Z}/p\mathbb{Z}$ or $k[x]/(x^r)$, where $k$ is a finite field). One reason to study such representation is that all continuous complex representations of the groups $\text{GL}_n(\mathbb{Z}/p\mathbb{Z})$ and $\text{GL}_n(k[[x]])$ (where $\mathbb{Z}/p\mathbb{Z}$ denotes the $p$-adic integers) arise from these finite quotients. I will explain why the natural generalization of the Harish-Chandra functors do not furnish a PSH algebra in this case, and how is this related to the Bruhat decomposition and Gauss elimination. In order to overcome this issue we have constructed a generalization of the Harish-Chandra functors. I will explain this generalization, describe some of the new functors properties, and explain how can they be applied to studying complex representations.

The talk will be based on a joint work with Tyrone Crisp and Uri Onn.