Geometric stability using information theory

Abstract:

Projection inequalities bound the volume of a body in terms of a product of the volumes of lower-dimensional projections of the body. Two well-known examples are the Loomis-Whitney inequality and the more general Uniform Cover inequality. We will see how to use information theory to prove stability versions of these inequalities, showing that when they are close to being tight, the body in question is close to being a box (which is the unique case of equality). We will also see how to obtain a stability result for the edge-isoperimetric inequality in the infinite d-dimensional lattice. Namely, that a subset of \( \mathbb{Z}^d \) with small edge-boundary must be close in symmetric difference to a d-dimensional cube.

Based on work with David Ellis, Ehud Friedgut and Guy Kindler.