Gibbs measures vs. random walks in negative curvature

Abstract:

The ideal boundary of a negatively curved manifold naturally carries two types of measures. On the one hand, we have conditionals for equilibrium (Gibbs) states associated to Hölder potentials; these include the Patterson-Sullivan measure and the Liouville measure. On the other hand, we have stationary measures coming from random walks on the fundamental group.

We compare and contrast these two classes. First, we show that both of these of these measures can be associated to geodesic flow invariant measures on the unit tangent bundle, with respect to which closed geodesics satisfy different equidistribution properties. Second, we show that the absolute continuity between a harmonic measure and a Gibbs measure is equivalent to a relation between entropy, (generalized) drift and critical exponent, generalizing previous formulas of Guivarc’h, Ledrappier, and Blachère-Haïssinsky-Mathieu. This shows that if the manifold (or more generally, a CAT(-1) quotient) is geometrically finite but not convex cocompact, stationary measures are always singular with respect to Gibbs measures.

A major technical tool is a generalization of a deviation inequality due to Ancona saying the so called Green distance associated to the random walk is nearly additive along geodesics in the universal cover.

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