Grassmann Graphs, Vertex Cover and 2-to-2 games

Abstract:

We discuss recent progress on hardness of 2-to-2 games, with applications to the inapproximability of the Vertex-Cover problem.

A 2-to-2 game (which is a variant of Khot's well known unique games), is defined by a graph where there is a variable in each node, and a constraint of a specific structure defined on each edge. While in unique games each edge-constraint must be a one-to-one correspondence -- i.e. for each assignment to one node there is exactly one assignment to the other node that satisfies the constraint -- in 2-to-2 games the correspondence has a "two-to-two" structure. The goal is to distinguish between instances in which almost all of the edge-constraints can be satisfied, and instances in which almost none of them can be satisfied simultaneously.

We present a new combinatorial hypothesis regarding Grassmann graphs, and show that it implies that 2-to-2 games are NP-hard *in a certain sense*. As a consequence, the hypothesis implies that it is NP-hard to distinguish between graphs that have an independent set of fractional size \((1-1/\sqrt{2})\), and graphs with no independent sets of any constant fractional size. This easily implies that it is NP-hard to approximate the Vertex Cover problem within a factor \(\sqrt{2} - o(1)\).

The talk is mostly based on a joint work with Subhash Khot and Muli Safra, nevertheless, we will also mention results from a more recent extension, which is a joint work with Irit Dinur, Subhash Khot, Guy Kindler and Muli Safra.