Harmonic Analysis on GL_n over finite fields

Abstract:

There are many formulas that express interesting properties of a finite group $G$ in terms of sums over its characters. For evaluating or estimating these sums, one of the most salient quantities to understand is the \textit{character ratio}:

\[
\frac{\text{trace}(\rho(g))}{\text{dim}(\rho)},
\]

for an irreducible representation $\rho$ of $G$ and an element $g$ of $G$. For example, Diaconis and Shahshahani stated a formula of the mentioned type for analyzing certain random walks on $G$.

Recently, we discovered that for classical groups $G$ over finite fields there is a natural invariant of representations that provides strong information on the character ratio. We call this invariant rank. This talk will discuss the notion of rank for $\text{GL}_n$ over finite fields, and explain how one can apply the results to verify mixing time and rate for certain random walks.

The talk will assume basic notions of linear algebra in Hilbert spaces, and the definition of a group.

This is joint work with Roger Howe (Yale and Texas AM).