On Herman’s positive metric entropy conjecture

Abstract:

We show that any area-preserving $C^r$-diffeomorphism of a two-dimensional surface displaying an elliptic fixed point can be $C^r$-perturbed to one exhibiting a chaotic island whose metric entropy is positive, for every $1 \leq r \leq 1$. This proves a conjecture of Herman stating that the identity map of the disk can be $C^\infty$-perturbed to a conservative diffeomorphism with positive metric entropy. This implies also that the Chirikov standard map for large and small parameter values can be $C^\infty$-approximated by a conservative diffeomorphisms displaying a positive metric entropy (a weak version of Sinai’s positive metric entropy conjecture). Finally, this sheds light onto a Herman’s question on the density of $C^r$-conservative diffeomorphisms displaying a positive metric entropy: we show the existence of a dense set formed by conservative diffeomorphisms which either are weakly stable (so, conjecturally, uniformly hyperbolic) or display a chaotic island of positive metric entropy. This is a joint work with Pierre Berger.