Abstract:
Symmetry is defined in terms of structure-preserving transformations (automorphisms); regularity in
terms of numerical invariants. Symmetry always implies regularity but there are many highly regular
combinatorial objects (such as "strongly regular graphs") with no symmetry. The opposite
of irregularity is regularity, not symmetry. Yet we show that in a well-defined sense, the opposite of
hidden irregularity is hidden symmetry, and in fact hidden symmetry of a particularly robust kind.
The symmetry of a circle is easily destroyed: just "individualize" two non-opposite points -- color one
of them red, the other blue -- and all the symmetry is gone. In fact, the resulting structure is
completely irregular: every point is uniquely identified by a pair of numerical invariants, namely, its
pair of distances to the two individualized points. We shall say that the circle has a high degree of
hidden irregularity.
In contrast, Johnson graphs are objects with robust symmetry: individualizing a small number of
vertices of a Johnson graph hardly makes a dent in its symmetry.
Recent work on the algorithmic problem of Graph Isomorphism has revealed that Johnson graphs are
unique in this regard: Every finite relational structure of small arity either has a measurable (say 10%)
hidden irregularity (revealed by individualizing a polylogarithmic number of elements) or has a large
degree of hidden symmetry, manifested in a canonically embedded Johnson graph on more than
90% of the underlying set.
This dichotomy is the key Divide-and-Conquer tool in recent progress on the worst-case complexity of
the Graph Isomorphism problem.
This subject is purely combinatorial and does not require advanced mathematical apparatus. The
group theoretic aspects of the new Graph Isomorphism test will be discussed in a follow-up seminar
on January 30.