Hitting Convex Sets with Points

Abstract:

Given an underlying finite point set \( P \) in the plane, we seek a small set \( Q \) that would hit any convex set that contains at least an \( \varepsilon \)-fraction of \( P \). Such a set \( Q \) is called a weak \( \varepsilon \)-net. The study of \( \varepsilon \)-nets is central to Computational and Combinatorial Geometry, and it bears important connections to Statistical Learning Theory, Extremal Combinatorics, Discrete Optimization, and other areas.

It is an outstanding open problem to determine tight asymptotic bounds on weak \( \varepsilon \)-nets with respect to convex sets. For any underlying point set in the plane we describe such a net whose cardinality is roughly proportional to \( \varepsilon^{-3/2} \). This is the first improvement of the over-25-year-old bound of Alon, Barany, Furedi, and Kleitman.