



THE WEIZMANN INSTITUTE OF SCIENCE  
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

Geometry and Topology Seminar & Mathematical Analysis and Applications Seminar

Room 290C ,Ziskind Building  
on Tuesday, Oct 17, 2017  
at 16:15

Gabriel Katz  
MIT

Holography of traversing flows and its applications to the inverse scattering  
problems

Abstract:

We study the non-vanishing gradient-like vector fields  $v$  on smooth compact manifolds  $X$  with boundary. We call such fields traversing. With the help of a boundary generic field  $v$ , we divide the boundary  $\partial X$  of  $X$  into two complementary compact manifolds,  $\partial^+X(v)$  and  $\partial^-X(v)$ . Then we introduce the causality map  $C_v: \partial^+X(v) \rightarrow \partial^-X(v)$ , a distant relative of the Poincare return map. Let  $\mathcal{F}(v)$  denote the oriented 1-dimensional foliation on  $X$ , produced by a traversing  $v$ -flow.

Our main result, the Holography Theorem, claims that, for boundary generic traversing vector fields  $v$ , the knowledge of the causality map  $C_v$  allows for a reconstruction of the pair  $(X, \mathcal{F}(v))$ , up to a homeomorphism  $\Phi: X \rightarrow X$  which is the identity on the boundary  $\partial X$ . In other words, for a massive class of ODE's, we show that the topology of their solutions, satisfying a given boundary value problem, is rigid. We call these results "holographic" since the  $(n+1)$ -dimensional  $X$  and the un-parameterized dynamics of the flow on it are captured by a single correspondence  $C_v$  between two  $n$ -dimensional screens,  $\partial^+X(v)$  and  $\partial^-X(v)$ .

This holography of traversing flows has numerous applications to the dynamics of general flows. Time permitting, we will discuss some applications of the Holography Theorem to the geodesic flows and the inverse scattering problems on Riemannian manifolds with boundary.