How Many Linear Equations are Required to Solve a Linear Program?

Abstract:

Linear Programming is the backbone of algorithm design and combinatorial optimization. The fastest known algorithms for solving linear programs, both in theory and practice, are based on Interior-Point Methods: The core idea behind IPMs is to iteratively use Newton approximation to reduce a linear program to a sequence of $\sim \sqrt{n}$ linear systems. Whether this number of iterations can be improved is one of the major open problems in convex optimization. A long line of work has shown that $\sqrt{n}$ is indeed optimal for the special class of *self-concordant* Barrier IPMs [Nestrov-Nemirovski94], but for general IPMs very little is known.

We propose a purely information-theoretic query model for studying the rate of convergence of IPMs, via linear-system queries: Each iteration of the algorithm can adaptively specify an arbitrary diagonal matrix $H$ (an ellipsoid) and a vector $v$, and the oracle returns the least-squares minimizer of the linear system $\arg\min_x \|Ax - v\|_H$. We show this model captures all known (deterministic) IPMs. Our main result is an $\Omega(\sqrt{n})$ lower bound on the iterations of any deterministic algorithm in this model, albeit for an (exponentially) ill-conditioned LP. In this talk I will describe this progress, assuming no prior knowledge on IPMs.

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