Improved Bounds for the Excluded Grid Theorem

Abstract:

We study the Excluded Grid Theorem, a fundamental structural result in graph theory, that was proved by Robertson and Seymour in their seminal work on graph minors. The theorem states that there is a function \( f \), such that for every integer \( g > 0 \), every graph of treewidth at least \( f(g) \) contains the \((gxg)\)-grid as a minor. For every integer \( g > 0 \), let \( f(g) \) be the smallest value for which the theorem holds. Establishing tight bounds on \( f(g) \) is an important graph-theoretic question. Robertson and Seymour showed that \( f(g) \) is at least of order \( g^2 \log g \). For a long time, the best known upper bounds on \( f(g) \) were super-exponential in \( g \). The first polynomial upper bound of \( f(g) = O(g^98 \text{ poly log } g) \) was proved by Chekuri and Chuzhoy. It was later improved to \( f(g) = O(g^{36} \text{ poly log } g) \), and then to \( f(g) = O(g^{19} \text{ poly log } g) \). In this talk we present our recent work that further improves this bound to \( f(g) = O(g^9 \text{ poly log } g) \) via a simpler proof. Moreover, while there are natural barriers that seem to prevent the previous methods from yielding tight bounds for the theorem, it seems conceivable that the techniques proposed in this thesis can lead to even tighter bounds on \( f(g) \).