A long-standing open problem, known as Hadwiger's covering problem, asks what is the smallest natural number $N(n)$ such that every convex body in $\mathbb{R}^n$ can be covered by a union of the interiors of at most $N(n)$ of its translates. Despite continuous efforts, the best general upper bound known for this number remains as it was more than sixty years ago, of the order of $\binom{2n}{n} \ln n$.

In this talk, I will discuss some history of this problem and present a new result in which we improve this bound by a sub-exponential factor. Our approach combines ideas from previous work, with tools from Asymptotic Geometric Analysis. As a key step, we use thin-shell estimates for isotropic log-concave measures to prove a new lower bound for the maximum volume of the intersection of a convex body $K$ with a translate of $-K$. We further show that the same bound holds for the volume of $K \cap (-K)$ if the center of mass of $K$ is at the origin.

If time permits we shall discuss some other methods and results concerning this problem and its relatives.

Joint work with H. Huang, B. Vritsiou, and T. Tkocz