Improved lower and upper bounds on the Hadwiger-Debrunner numbers

Abstract:

A family of sets F is said to satisfy the \((p,q)\)-property if among any \(p\) sets in F, some \(q\) have a non-empty intersection. Hadwiger and Debrunner (1957) conjectured that for any \(p > q > d\) there exists a constant \(c = c_d(p,q)\), such that any family of compact convex sets in \(R^d\) that satisfies the \((p,q)\)-property, can be pierced by at most \(c\) points. The classical Helly's Theorem is equivalent to the fact that \(c_d(p,p)=1\) (\(p > d\)).

In a celebrated result from 1992, Alon and Kleitman proved the conjecture. However, obtaining sharp bounds on the minimal such \(c_d(p,q)\), called `the Hadwiger-Debrunner numbers', is still a major open problem in combinatorial geometry.

In this talk we present improved upper and lower bounds on the Hadwiger-Debrunner numbers, the latter using the hypergraph container method.

Based on joint works with Shakhar Smorodinsky and Gabor Tardos.