The index theorem for self-adjoint elliptic operators with local boundary conditions

Abstract:

The spectral flow is a well-known invariant of a 1-parameter family of self-adjoint Fredholm operators. It is defined as the net number of operator’s eigenvalues passing through 0 with the change of parameter.

Let S be a compact surface with non-empty boundary. Consider the space Ell(S) of first order self-adjoint elliptic differential operators on S with local boundary conditions. The first part of the talk is devoted to the computing of the spectral flow along loops in Ell(S), and also along paths with conjugated ends.

After that we consider more general situation: a family of elements of Ell(S) parameterized by points of a compact space X. We define the topological index of such a family and show that it coincides with the analytical index of the family. Both indices take value in $K^1(X)$. When $X$ is a circle, this result turns into the formula for the spectral flow from the first part of the talk.