The index theorem for self-adjoint elliptic operators with local boundary conditions

Abstract:

The spectral flow is a well-known invariant of a 1-parameter family of self-adjoint Fredholm operators. It is defined as the net number of operator’s eigenvalues passing through 0 with the change of parameter.

Let \( S \) be a compact surface with non-empty boundary. Consider the space \( \text{Ell}(S) \) of first order self-adjoint elliptic differential operators on \( S \) with local boundary conditions. The first part of the talk is devoted to the computing of the spectral flow along loops in \( \text{Ell}(S) \), and also along paths with conjugated ends.

After that we consider more general situation: a family of elements of \( \text{Ell}(S) \) parameterized by points of a compact space \( X \). We define the topological index of such a family and show that it coincides with the analytical index of the family. Both indices take value in \( K^1(X) \). When \( X \) is a circle, this result turns into the formula for the spectral flow from the first part of the talk.