Introduction to derived algebraic and analytic geometry

Abstract:

I will present a 'categorical' way of doing analytic geometry in which analytic geometry is seen as a precise analogue of algebraic geometry. Our approach works for both complex analytic geometry and p-adic analytic geometry in a uniform way. I will focus on the idea of an 'open set' as used in these various areas of math and how it is characterised categorically. In order to do this, we need to study algebras and their modules in the category of Banach spaces. The categorical characterization that we need uses homological algebra in these 'quasi-abelian' categories which is work of Schneiders and Prosmans. In fact, we work with the larger category of Ind-Banach spaces for reasons I will explain. This gives us a way to establish foundations of analytic geometry and to compare with the standard notions such as the theory of affinoid algebras, Grosse-Klønn's theory of dagger algebras (overconvergent functions), the theory of Stein domains and others. I will explain how this extends to a formulation of derived analytic geometry following the relative algebraic geometry approach of Toen, Vaquie and Vezzosi.

This is joint work with Federico Bambozzi (Regensburg) and Kobi Kremnizer (Oxford).