Invertibility of sparse random matrices

Abstract:

We consider a class of sparse random matrices of the form $A_n = (\xi_{i,j} \delta_{i,j})_{i,j=1}^n$, where $\{\xi_{i,j}\}$ are i.i.d.~centered random variables, and $\{\delta_{i,j}\}$ are i.i.d.~Bernoulli random variables taking value $1$ with probability $p_n$, and prove a quantitative estimate on the smallest singular value for $p_n = \Omega(\frac{\log n}{n})$, under a suitable assumption on the spectral norm of the matrices. This establishes the invertibility of a large class of sparse matrices. We also find quantitative estimates on the smallest singular value of the adjacency matrix of a directed Erdos-Reyni graph whenever its edge connectivity probability is above the critical threshold $\Omega(\frac{\log n}{n})$. This is joint work with Mark Rudelson.