Isolated hypersurface singularities and associated forms

Abstract:

In our recent articles joint with M. Eastwood and J. Alper, it was conjectured that all rational $GL_n$-invariant functions of forms of degree $d>2$ on complex space $C^n$ can be extracted, in a canonical way, from those of degree $n(d-2)$ by means of assigning every form with nonvanishing discriminant the so-called associated form. While this surprising statement is interesting from the point of view of classical invariant theory, its original motivation was the reconstruction problem for isolated hypersurface singularities, which is the problem of finding a constructive proof of the well-known Mather-Yau theorem. Settling the conjecture is part of our program to solve the reconstruction problem for quasihomogeneous isolated hypersurface singularities. This amounts to showing that a certain system of invariants arising from the Milnor algebras of such singularities is complete, and the conjecture implies completeness in the homogeneous case. In my talk I will give an overview of the recent progress. If time permits, I will further discuss the map that assigns a non-degenerate form its associated form. This map is rather natural and deserves attention regardless of the conjecture. For instance, it induces a natural equivariant involution on the space of elliptic curves with non-vanishing j-invariant. Surprisingly, the existence of such an involution appears to be a new fact.