L$_2$ Mixing and hypercontractivity via maximal inequalities and hitting-times

Abstract:

There are numerous essentially equivalent characterizations of mixing in $L_1$ of a finite Markov chain. Some of these characterizations involve natural probabilistic concepts such as couplings, stopping times and hitting times. In contrast, while there are several analytic and geometric tools for bounding the $L_2$ mixing time, none of them are tight and they do not have a probabilistic interpretation.

We provide tight probabilistic characterizations in terms of hitting times distributions for mixing in $L_2$ (for normal chains) and (under reversibility) in relative entropy. This is done by assigning appropriate penalty (depending on the size of the set) to the case that the chain did not escape from a certain set.

We also prove a new extremal characterization of the log-sobolev constant in terms of a weighted version of the spectral gap (where the weight depends on the size of the support of the function).