The talk will focus on the case of macroscopic statistics of the spectrum of Hermitian matrices - in particular Wigner matrices - as the empirical distribution of the eigenvalues, the largest eigenvalue or the traces of powers.

In a first part, I will be concerned with the so-called "objective method". Coined by David Aldous, this method consists in introducing, given a sequence of random objects, like random finite graphs, a new infinite random object from which one can deduce asymptotic properties of the original sequence. In the context of random matrices, this method has been mainly advertised by Balint Virag, and proven effective in showing universality results for the so-called beta-ensembles. Regarding large deviations of random matrices, this "objective method" amounts to embed our sequence of matrices with growing size into an appropriate space on which one is able to understand the large deviations, and carry out a contraction principle. I will review several large deviations principles obtained by this method, given by interpretations of random matrices as either dense or sparse graphs, and point out the limits of this strategy.