Limit cycle enumeration for random vector fields

Abstract:

The second part of Hilbert's sixteenth problem asks for a study of the number and relative positions of the limit cycles of an ODE system associated to a planar vector field with polynomial component functions. Seeking a probabilistic perspective on Hilbert's problem, we present recent results on the distribution of limit cycles when the vector field component functions are random polynomials. We present a lower bound for the average number of limit cycles for the Kostlan-Shub-Smale model, and we present asymptotic results for a special class of limit cycle enumeration problems concerning a randomly perturbed center focus. We will discuss some ideas from the proofs which combine techniques from dynamical systems (such as transverse annuli, Poincaré maps, and Melnikov functions) with those coming from the theory of random analytic functions (such as real zeros of random series, the Kac-Rice formula, and the barrier construction from the study of nodal sets of random eigenfunctions). We conclude by discussing future directions and open problems.

Relevant text: preprint on arXiv