For a uniform random labelled tree, we find the limiting distribution of tree parameters which are stable (in some sense) with respect to local perturbations of the tree. The proof is based on the martingale central limit theorem and the Aldous Broder algorithm. In particular, our general result implies the asymptotic normality of the number of occurrences of any given small pattern and the asymptotic log-normality of the number of automorphisms.