Linear algorithms for computing conformal mappings

Abstract:

(joint with Noam Aigerman, Raz Sluzky and Yaron Lipman)
Computing homeomorphisms between surfaces is an important task in shape analysis fields such as computer graphics, medical imaging and morphology. A fundamental tool for these tasks is solving Dirichlet's problem on an arbitrary Jordan domain with disc topology, where the boundary of the domain is mapped homeomorphically to the boundary of a specific target domain: A convex polygon. By the Rado-Kneser-Choquet Theorem such harmonic mappings are homeomorphisms onto the convex polygon. Standard finite element approximations of harmonic mappings lead to discrete harmonic mappings, which have been proven to be homeomorphisms as well. Computing the discrete harmonic mappings is very efficient and reliable as the mappings are obtained as the solution of a sparse linear system of equations.

In this talk we show that the methodology above, can be used to compute *conformal* homeomorphisms, for domains with either disc or sphere topology:

By solving Dirichlet's problem with correct boundary conditions, we can compute conformal homeomorphisms from arbitrary Jordan domains to a specific canonical domain- a triangle. The discrete conformal mappings we compute are homeomorphisms, and approximate the conformal homeomorphism uniformly and in $H^1$. Similar methodology can also be used to conformally map a sphere type surface to a planar Jordan domain, whose edges are identified so that the planar domain has the topology of a sphere.