Long-range order in random 3-colorings of \( \mathbb{Z}^d \)

Abstract:

Consider a random coloring of a bounded domain in \( \mathbb{Z}^d \) with the probability of each coloring \( F \) proportional to \( \exp(-\beta \times N(F)) \), where \( \beta > 0 \) is a parameter (representing the inverse temperature) and \( N(F) \) is the number of nearest neighboring pairs colored by the same color. This is the anti-ferromagnetic 3-state Potts model of statistical physics, used to describe magnetic interactions in a spin system. The Kotecký conjecture is that in such a model, for \( d \geq 3 \) and high enough \( \beta \), a sampled coloring will typically exhibit long-range order, placing the same color at most of either the even or odd vertices of the domain. We give the first rigorous proof of this fact for large \( d \). This extends previous works of Peled and of Galvin, Kahn, Randall and Sorkin, who treated the case \( \beta = \infty \).

The main ingredient in our proof is a new structure theorem for 3-colorings which characterizes the ways in which different "phases" may interact, putting special emphasis on the role of edges connecting vertices of the same color. We also discuss several related conjectures. No background in statistical physics will be assumed and all terms will be explained thoroughly.

Joint work with Ohad Feldheim.