Low Distortion Embedding from Edit to Hamming Distance using Coupling

Abstract:

The Hamming and the edit metrics are two common notions of measuring distances between pairs of strings $x,y$ lying in the Boolean hypercube. The edit distance between $x$ and $y$ is defined as the minimum number of character insertion, deletion, and bit flips needed for converting $x$ into $y$. Whereas, the Hamming distance between $x$ and $y$ is the number of bit flips needed for converting $x$ to $y$.

In this paper we study a randomized injective embedding of the edit distance into the Hamming distance with a small distortion. This question was studied by Jowhari (ESA 2012) and is mainly motivated by two questions in communication complexity: the document exchange problem and deciding edit distance using a sketching protocol.

We show a randomized embedding with quadratic distortion. Namely, for any $x,y$ satisfying that their edit distance equals $k$, the Hamming distance between the embedding of $x$ and $y$ is $O(k^2)$ with high probability. This improves over the distortion ratio of $O(\log n \log^{\ast} n)$ obtained by Jowhari for small values of $k$. Moreover, the embedding output size is linear in the input size and the embedding can be computed using a single pass over the input.