Low Distortion Embedding from Edit to Hamming Distance using Coupling

Abstract:

The Hamming and the edit metrics are two common notions of measuring distances between pairs of strings \(x, y\) lying in the Boolean hypercube. The edit distance between \(x\) and \(y\) is defined as the minimum number of character insertion, deletion, and bit flips needed for converting \(x\) into \(y\). Whereas, the Hamming distance between \(x\) and \(y\) is the number of bit flips needed for converting \(x\) to \(y\).

In this paper we study a randomized injective embedding of the edit distance into the Hamming distance with a small distortion. This question was studied by Jowhari (ESA 2012) and is mainly motivated by two questions in communication complexity: the document exchange problem and deciding edit distance using a sketching protocol.

We show a randomized embedding with quadratic distortion. Namely, for any \(x, y\) satisfying that their edit distance equals \(k\), the Hamming distance between the embedding of \(x\) and \(y\) is \(O(k^2)\) with high probability. This improves over the distortion ratio of \(O(\log n \log^* n)\) obtained by Jowhari for small values of \(k\). Moreover, the embedding output size is linear in the input size and the embedding can be computed using a single pass over the input.