Meromorphic differentials with prescribed singularities: a refinement of a classical Mittag-Leffler theorem

Abstract:

A classical theorem of Mittag-Leffler asserts that in a given Riemann surface $X$, for any pattern of multiplicities of poles and any configuration of residues (summing to zero), there is a meromorphic 1-form on $X$ that realizes them. The only obstruction is that residues at simple poles should be nonzero.

If we require that the multiplicity of the zeroes is also prescribed, the problem can be reformulated in terms of strata of meromorphic differentials. Using the dictionary between complex analysis and flat geometry, we are able to provide a complete characterization of configurations of residues that are realized for a given pattern of singularities. Two nontrivial obstructions appear concerning the combinatorics of the multiplicity of zeroes and the arithmetics of the residues. This is a joint work with Quentin Gendron.