Consider a random finite metric space $X$ given by sampling $n$ points in the unit interval uniformly, and a deterministic finite metric space $U$ given by placing $n$ points in the unit interval at uniform distance. With high probability, $X$ will contain some pairs of points at distance roughly $1/n^2$, so any bijection from $X$ to $U$ must distort distances by a factor of roughly $n$. However, with high probability, two of these random spaces, $X_1$ and $X_2$, have a bijection which distorts distances by a factor of only about $n^{2/3}$. The exponent of $2/3$ is optimal.