Metric entropies, $L_1$ transportation, and competitive analysis

Abstract:

The MTS problem (Borodon, Linial, and Saks 1992) is a general model for the analysis of algorithms that optimize in the presence of information arriving over time, where the state space is equipped with a metric. I will discuss a relatively new approach to this area, where both the algorithm and method of analysis are derived canonically from a choice of a “regularizer” on the probability simplex. The regularizer can be interpreted as a Riemannian structure on the simplex, and then the algorithm is simply a gradient flow.

Defining the regularizer as an appropriate “noisy” multiscale metric entropy (similar to, e.g., the Talagrand $\gamma_1$ functional) yields the best-known competitive ratio for every metric space. For ultrametrics (aka, “HST metrics”), this achieves the conjectured bound, but the algorithm falls short of resolving the MTS conjecture for many other spaces, including subsets of the real line.